

DISCUSSION PAPER SERIES

Discussion paper No. 170

Trade, Productivity and Welfare when Monopolistic Competition and Oligopoly Coexist

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January 2018



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TRADE, PRODUCTIVITY AND WELFARE WHEN MONOPOLISTIC COMPETITION AND OLIGOPOLY COEXIST

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Abstract

We develop a two-country general equilibrium model where monopolistically competitive and oligopolistic industries coexist, and intra-firm division of labor involves economies of scale. If market size increases, the productivity of all industries and welfare improve. However, as the proportion of trading sectors rises, the productivity of trading industries increases, but that of non-trading industries decreases. Although the welfare effect of expansion of trading sectors is analytically unclear, a numerical simulation tells that it is positive.

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I. INTRODUCTION

Recent evidence on international trade has reached one stylized fact that ‘engaging in international trade is an exceedingly rare activity.’ (Bernard et al., 2007, p. 105) According to Bernard et al. (2007), only 4% of the US firms exported in 2000. Mayer and Ottaviano (2008) and Freund and Pierola (2015) also find similar evidence for the 7 western European countries and 32 developing countries, respectively.¹ This stylized fact highlights the importance of a small number of large firms (superstar firms) in international trade today, and it is required to depart from the monopolistic competition model with massless firms.² The reality is well described by a coexistence of monopolistic competition and oligopoly.

In order to incorporate the above recognition, we develop a two-country model that has the following features. First, we assume a continuum of industries, some of which are monopolistically competitive and the others of which are oligopolistic. Second, we allow both trading and non-trading sectors. Third, following the formulation of division of labor in Chaney and Ossa (2013), we stress the productivity effect of trade.³ In this model, we define trade liberalization in two ways; an increase in market size like Krugman (1979) and Chaney and Ossa (2013) and an increase in the proportion of trading sectors like Bastos and Straume (2012) and Kreickemeier and Meland (2013). We show that market size expansion raises the productivity of all industries and welfare, which is a straightforward extension of Chaney and Ossa (2013). However, the effects of an increase in the share of trading sectors are complex. It raises the productivity of trading industries, but lowers that of non-trading industries. Furthermore, its welfare effect is analytically ambiguous. However, a numerical simulation suggests that it raises welfare.

¹More recent empirical studies are mentioned in Section V.

²To our knowledge, Neary (2004) is the first to point out the limitation of the monopolistic competition model in international trade.

³Because the threshold between trading and non-trading sectors is assumed exogenous, the reallocation effect of Melitz (2003) is assumed away.

This paper is influenced by two strands of literature. The first concerns a general oligopolistic equilibrium (GOLE) model of Neary (2003, 2016).⁴ Assuming a continuum of oligopolistic industries and supposing that ‘oligopolistic firms should be modeled as large in their own markets but small in the economy as a whole,’ (Neary, 2016, p. 687) he presents a consistent and tractable model of oligopoly that has many applications. While the existing papers employing this approach assume an oligopoly in all sectors, we introduce monopolistically competitive sectors. The second related literature is about the coexistence of monopolistic competition and oligopoly. Shimomura and Thisse (2012) first characterize such a mixed market in a closed economy. Then, Parenti (2018) uses a slightly different model, and considers the effect of trade.⁵ Parenti (2018) shows that trade liberalization necessarily improves welfare.⁶ Vavoura (2017) also obtains the similar result in a CES preference model. While these authors assume that monopolistically competitive and oligopolistic *firms* coexist in a single industry, we consider a different situation in which monopolistically competitive and oligopolistic *industries* coexist.

This paper is organized as follows. Section II presents a model. Sections III and IV address the effects of trade on the productivities and welfare, respectively. Section V offers some discussions about our model and results. Section VI concludes.

II. MODEL

This section presents a model. Suppose two identical countries (Home and Foreign), two industries (monopolistic competition and Cournot com-

⁴The working paper version of Neary (2016) was released in 2002. Colacicco (2015) is a comprehensive survey.

⁵The utility function in Shimomura and Thisse (2012) is CES, but that in Parenti (2018) is quadratic.

⁶Parenti (2018) defines trade liberalization in two ways; an increase in the number of trading countries and a reduction in trade cost.

petition), and one factor (labor). There is a continuum of goods on a unit interval, and \bar{z} denotes the threshold that divides a set of monopolistically competitive goods and a set of oligopolized goods; good z is supplied under monopolistic competition for $z \in [0, \bar{z}]$ and under oligopoly for $z \in [\bar{z}, 1]$. In addition, \tilde{z} represents a proportion of trading industries. Thus, the whole economy comprises four kinds of goods. If $z \in [0, \tilde{z}\bar{z}]$, the good is a tradable produced under monopolistic competition, $z \in [\tilde{z}\bar{z}, \bar{z}]$, the good is a non-tradable produced under monopolistic competition, $z \in [\bar{z}, \bar{z} + \tilde{z}(1 - \bar{z})]$, the good is a tradable produced under oligopoly, and if $z \in [\bar{z} + \tilde{z}(1 - \bar{z}), 1]$, the good is a non-tradable produced under oligopoly.

While this model contains many notations, we explain the process of division of labor, consumer behavior, firm behavior in monopolistic competition, firm behavior in an oligopoly, and market-clearing in the labor market in order.

a) Division of labor

As in Chaney and Ossa (2013), a production process involves division of labor.⁷ In order to produce one unit of final good, each firm performs a series of tasks on a closed interval $[0, 2]$. If one task, say, $\omega_1 \in (0, 2)$ is completed, an intermediate good ω_1 is produced. Then, ω_1 is used for the production of the next task, say, $\omega_2 \in (\omega_1, 2)$ by performing a task $\omega \in [\omega_1, \omega_2]$. Letting c be a core competency associated with each task, labor demand needed for performing one unit of task $\omega \in [\omega_1, \omega_2]$ is assumed to be given by

$$l(\omega_1, \omega_2) = \frac{1}{2} \int_{\omega_1}^{\omega_2} |c - \omega|^\gamma d\omega, \quad \gamma > 0.$$

In addition, each team that performs a task has to input fixed amount of

⁷See Chaney and Ossa (2013, p. 178) for a more detailed explanation.

labor $f > 0$. Given these assumptions, per-firm total cost becomes

$$\text{total cost} = wt \left(f + y \int_0^{\frac{1}{\gamma}} \omega^\gamma d\omega \right) = w \left(tf + \frac{yt^{-\gamma}}{1+\gamma} \right),$$

where w is the wage rate, t is the number of teams, and y is output. Minimizing this cost with respect to t , the optimal number of teams is obtained as

$$t = \left[\frac{\gamma y}{(1+\gamma)f} \right]^{\frac{1}{1+\gamma}}. \quad (1)$$

Throughout this paper, we call this cost-minimizing number of teams ‘firm productivity.’ Substituting (1) into total cost yields

$$\text{total cost} = w \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} y^{\frac{1}{1+\gamma}},$$

that is, ‘the production technology exhibits increasing returns to scale.’ (Chaney and Ossa, p. 178)

b) Preference and demand

We now consider utility maximization of consumers, and derive the demand functions of each category of goods. There are L identical consumers in each country whose preference is

$$U = \int_0^{\tilde{z}\tilde{z}} \ln X_1(z) dz + \int_{\tilde{z}\tilde{z}}^{\tilde{z}} \ln \tilde{X}_1(z) dz + \int_{\tilde{z}}^{\tilde{z}+\tilde{z}(1-\tilde{z})} \ln X_2(z) dz + \int_{\tilde{z}+\tilde{z}(1-\tilde{z})}^1 \ln \tilde{X}_2(z) dz \quad (2)$$

$$X_1(z) \equiv \left[\sum_{i=1}^m x_i(z)^\theta + \sum_{i=1}^m x_i^*(z)^\theta \right]^{\frac{1}{\theta}}, \quad \tilde{X}_1(z) \equiv \left[\sum_{i=1}^{\tilde{m}} \tilde{x}_i(z)^\theta \right]^{\frac{1}{\theta}}, \quad (3)$$

where U is utility, $X_1(z)$, $x_i(z)$ and $X_2(z)$ are a per-capita quantity index of monopolistically competitive tradables, consumption of them and consumption of oligopolized tradables, respectively. And, m is the number of traded

varieties. A tilde denotes the counterparts that are non-traded, and an asterisk is attached to a Foreign variable. Consumers choose consumption to maximize (2) under the budget constraint:

$$\begin{aligned} & \int_0^{\tilde{z}\bar{z}} \left[\sum_{i=1}^m p_i(z)x_i(z) + \sum_{i=1}^m p_i^*(z)x_i^*(z) \right] dz + \int_{\tilde{z}\bar{z}}^{\bar{z}} \left[\sum_{i=1}^{\tilde{m}} \tilde{p}_i(z)\tilde{x}_i(z) \right] dz \\ & + \int_{\bar{z}}^{\bar{z}+\tilde{z}(1-\bar{z})} P_2(z)X_2(z)dz + \int_{\bar{z}+\tilde{z}(1-\bar{z})}^1 \tilde{P}_2(z)\tilde{X}_2(z)dz \leq I, \end{aligned} \quad (4)$$

where $p_i(z)$ and $P_2(z)$ are a price of each monopolistically competitive and oligopolized tradable, respectively, and I is (nominal) income. The usage of an asterisk and tilde is the same as above.

In deriving the demand function of each good, we employ Neary's (2003, 2016) approach. According to him, consumers and all firms including oligopolistic firms take as given the marginal utility of income (Lagrangian multiplier associated with the budget constraint) since they are small in the whole economy. By making this assumption, Neary (2003, 2016) shows many interesting results on the welfare effect of competition policy and international trade, without worrying about the problems arising in general equilibrium oligopoly models. We also adopt the same assumption, namely, marginal utility of income serves as a numeraire and is normalized to unity. Then, solving the first-order conditions for utility maximization, the demand function of each good is obtained as follows.

$$x_i(z) = \frac{p_i(z)^{\frac{1}{\theta-1}}}{\sum_{i=1}^m p_i(z)^{\frac{\theta}{\theta-1}} + \sum_{i=1}^m p_i^*(z)^{\frac{\theta}{\theta-1}}}, \quad x_i^*(z) = \frac{p_i^*(z)^{\frac{1}{\theta-1}}}{\sum_{i=1}^m p_i(z)^{\frac{\theta}{\theta-1}} + \sum_{i=1}^m p_i^*(z)^{\frac{\theta}{\theta-1}}} \quad (5)$$

$$\tilde{x}_i(z) = \frac{\tilde{p}_i(z)^{\frac{1}{\theta-1}}}{\sum_{i=1}^{\tilde{m}} \tilde{p}_i(z)^{\frac{\theta}{\theta-1}}} \quad (6)$$

$$X_2(z) = \frac{1}{P_2(z)}, \quad \tilde{X}_2(z) = \frac{1}{\tilde{P}_2(z)}. \quad (7)$$

Since there are $L + L = 2L$ identical consumers in the world, the market-clearing condition of each good becomes

$$2Lx_i(z) = \frac{2Lp_i(z)^{\frac{1}{\theta-1}}}{\sum_{i=1}^m p_i(z)^{\frac{\theta}{\theta-1}} + \sum_{i=1}^m p_i^*(z)^{\frac{\theta}{\theta-1}}} \quad (8)$$

$$2Lx_i^*(z) = \frac{2Lp_i^*(z)^{\frac{1}{\theta-1}}}{\sum_{i=1}^m p_i(z)^{\frac{\theta}{\theta-1}} + \sum_{i=1}^m p_i^*(z)^{\frac{\theta}{\theta-1}}} \quad (9)$$

$$L\tilde{x}_i(z) = \frac{L\tilde{p}_i(z)^{\frac{1}{\theta-1}}}{\sum_{i=1}^{\tilde{m}} \tilde{p}_i(z)^{\frac{\theta}{\theta-1}}} \quad (10)$$

$$\frac{2L}{P_2(z)} = \sum_{j=1}^n y_j(z) + \sum_{j=1}^n y_j^*(z), \quad \frac{L}{\tilde{P}_2(z)} = \sum_{j=1}^n \tilde{y}_j(z),$$

where $n \geq 2$ is the number of oligopolistic firms, $y_j(z)$ and $y_j^*(z)$ are output of Home and Foreign oligopolistic firms that export, respectively, and $\tilde{y}_j(z)$ is output of non-trading oligopolists. Solving the last two equations for $P_2(z)$ and $\tilde{P}_2(z)$, the inverse demand function of oligopolized goods is given by

$$P_2(z) = \frac{2L}{\sum_{j=1}^n y_j(z) + \sum_{j=1}^n y_j^*(z)}, \quad \tilde{P}_2(z) = \frac{L}{\sum_{j=1}^n \tilde{y}_j(z)}. \quad (11)$$

Given the demand and inverse demand functions above, the firm profit in each category of industries is defined by

$$\begin{aligned} \pi_{1i}(z) &\equiv 2Lp_i(z)x_i(z) - w \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [2Lx_i(z)]^{\frac{1}{1+\gamma}} \\ \tilde{\pi}_{1i}(z) &\equiv L\tilde{p}_i(z)\tilde{x}_i(z) - w \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [L\tilde{x}_i(z)]^{\frac{1}{1+\gamma}} \\ \pi_{2j}(z) &\equiv P_2(z)y_j(z) - w \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [y_j(z)]^{\frac{1}{1+\gamma}} \\ \tilde{\pi}_{2j}(z) &\equiv \tilde{P}_2(z)\tilde{y}_j(z) - w \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [\tilde{y}_j(z)]^{\frac{1}{1+\gamma}} \end{aligned}$$

In what follows, profit maximization in monopolistically and oligopolistically competitive sectors is formalized.

c) Monopolistic competition

Each monopolistically competitive firm chooses price to maximize profit, given the market demand function in (8), (9) and (10).⁸ Then, the markup pricing rule is derived:

$$\begin{aligned} p_i(z) &= \frac{w}{(1+\gamma)\theta} \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [2Lx_i(z)]^{\frac{1}{1+\gamma}-1} \\ \tilde{p}_i(z) &= \frac{w}{(1+\gamma)\theta} \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [L\tilde{x}_i(z)]^{\frac{1}{1+\gamma}-1}. \end{aligned}$$

And, since free entry and exit drives profit to zero, price must be equal to average cost:

$$\begin{aligned} p_i(z) &= w \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [2Lx_i(z)]^{\frac{1}{1+\gamma}-1} \\ \tilde{p}_i(z) &= w \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} [L\tilde{x}_i(z)]^{\frac{1}{1+\gamma}-1} \end{aligned} \quad (12)$$

At this stage, we make a useful but admittedly artificial assumption. Specifically, let us suppose that θ depends on per-capita consumption of a differentiated good like $\theta(x_i)$ and $\theta(\tilde{x}_i)$. That is, consumers and firms take θ as given in maximizing utility and profit while it is a function of per-capita consumption. One justification is that the markup in Chaney and Ossa (2013) also depends on per-capita consumption.⁹ Considering that the

⁸The result is obtained even if output is chosen.

⁹Chaney and Ossa (2013) assume a non-CES utility function

$$U = \sum_{i=1}^m u(x_i) + \sum_{i=1}^m u(x_i^*),$$

which follows Krugman (1979). Under this preference, the markup becomes $\epsilon(x)/[\epsilon(x)-1]$, where $\epsilon(x) \equiv -u'(x)/[xu''(x)]$.

markup in our model is $1/\theta$, it may be fair to assume that θ depends on per-capita consumption x as in Krugman (1979) and Chaney and Ossa (2013). This variable markup is empirically well recognized.¹⁰

If one accepts this assumption, dividing the markup pricing rule by the zero profit condition yields

$$(1 + \gamma)\theta(x_i(z)) = (1 + \gamma)\theta(\tilde{x}_i(z)) = 1.$$

Since this equation holds for all differentiated goods, per-capita consumption is uniquely determined in the above equation, and it holds that $x_i(z) = \bar{x}_i(z) = x$ for all i and z . Substituting this result into the market-clearing conditions, we have

$$2Lx = \frac{2L}{2mp_i(z)}, \quad Lx = \frac{L}{\tilde{m}\tilde{p}_i(z)},$$

and hence the number of varieties is derived as follows.

$$m = \frac{1}{2p_i(z)x}, \quad \tilde{m} = \frac{1}{\tilde{p}_i(z)x}, \quad (13)$$

where $p_i(z)$ and $\tilde{p}_i(z)$ are given by (12) with $x_i(z)$ and $\tilde{x}_i(z)$ replaced by x .

d) Oligopoly

Each oligopolist chooses output to maximize profits in a Cournot fashion. Solving the system of the first-order conditions for profit maximization, the Cournot equilibrium outputs are obtained by

$$y = \left[\frac{(2n-1)(1+\gamma)L}{2n^2w} \right]^{1+\gamma} \left[\frac{\gamma}{(1+\gamma)f} \right]^\gamma, \quad \tilde{y} = \left[\frac{(n-1)(1+\gamma)L}{n^2w} \right]^{1+\gamma} \left[\frac{\gamma}{(1+\gamma)f} \right]^\gamma, \quad (14)$$

where subscript j and argument z are suppressed because all firms produce the same amount.

¹⁰See, for example, Edmond et al. (2015), Lu and Yu (2015) and De Loecker et al. (2016)

e) General equilibrium

Having characterized the equilibrium in each industry, we now close the model by introducing the labor market-clearing condition:

$$\begin{aligned}
L &= \int_0^{\tilde{z}\tilde{z}} m \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} (2Lx)^{\frac{1}{1+\gamma}} dz + \int_{\tilde{z}\tilde{z}}^{\tilde{z}} \tilde{m} \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} (Lx)^{\frac{1}{1+\gamma}} dz \\
&\quad + \int_{\tilde{z}}^{\tilde{z}+\tilde{z}(1-\tilde{z})} n \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} y^{\frac{1}{1+\gamma}} + \int_{\tilde{z}+\tilde{z}(1-\tilde{z})}^1 n \left[\frac{(1+\gamma)f}{\gamma} \right]^{\frac{\gamma}{1+\gamma}} \tilde{y}^{\frac{1}{1+\gamma}} dz \\
&= \frac{\tilde{z}L}{w} + \frac{(1-\tilde{z})(1+\gamma)L[(2n-1)\tilde{z} + 2(n-1)(1-\tilde{z})]}{2nw},
\end{aligned}$$

where L , which stands for the number of consumers, represents the labor endowment since each consumer inelastically supplies one unit of labor in the left-hand side. The last equality follows by substituting m and \tilde{m} in (13) and y and \tilde{y} in (14). Solving this equation for w , the equilibrium wage rate is determined by

$$w = \frac{2n\tilde{z} + (1-\tilde{z})(1+\gamma)[(2n-1)\tilde{z} + 2(n-1)(1-\tilde{z})]}{2n}. \quad (15)$$

Once the wage rate is derived, all the other endogenous variables are also obtained. From (15), we see that:

Proposition 1: Market size L has no effect on the equilibrium wage, but the wage rate rises with the portion of trading industries \tilde{z} .

Proof. The former part is trivial, and the latter part is proved by differentiating (15) with respect to \tilde{z} :

$$\frac{\partial w}{\partial \tilde{z}} = \frac{(1-\tilde{z})(1+\gamma)}{2n} > 0.$$

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The intuition behind this result is as follows. Since both labor demand and supply are proportionate with L , namely, L is multiplied on both the left-

and right-hand sides in the labor market-clearing condition, the equilibrium wage is not affected by L . If, in contrast, \tilde{z} increases, labor demand from trading industries expands and that from non-trading industries contracts. However, output and labor demand of trading industries is larger than those of non-trading industries, and hence the former effect outweighs the latter effect, resulting in a higher wage rate. This result is not interesting per se, but it will play an important role in interpreting the effects of trade on productivities and welfare.

In the variant of the two-country GOLE model with linear demand and no monopolistically competitive sector, Bastos and Straume (2012) and Kreckemeier and Meland (2013) also establish that the competitive wage rate rises with the proportion of trading industries. In addition, assuming that all industries are oligopolistic, Fujiwara and Kamei (2018) arrive at the same result.¹¹ In contrast, one can show that if all industries are monopolistically competitive, a change in \tilde{z} has no effect on the wage rate. This is because an increase in \tilde{z} raises labor demand in the trading industries, but lowers labor demand in the non-trading industries by exactly the same amount.

III. PRODUCTIVITY EFFECT

This section addresses the effect of trade on the productivity. Following Chaney and Ossa (2013) and Kamei (2014), we refer to the optimal number of teams in (1) as firm productivity. Then, firm productivity of each monopolistically competitive firm and oligopolistic firm is respectively obtained by

$$t_1 = \left[\frac{2L\gamma x}{(1+\gamma)f} \right]^{\frac{1}{1+\gamma}}, \quad \tilde{t}_1 = \left[\frac{L\gamma x}{(1+\gamma)f} \right]^{\frac{1}{1+\gamma}} \quad (16)$$

$$t_2 = \left[\frac{\gamma y}{(1+\gamma)f} \right]^{\frac{1}{1+\gamma}}, \quad \tilde{t}_2 = \left[\frac{\gamma \tilde{y}}{(1+\gamma)f} \right]^{\frac{1}{1+\gamma}}, \quad (17)$$

¹¹This case is easily reproduced by substituting $\bar{z} = 0$ in our model.

where t_1 is firm productivity of a trading firm in monopolistic competition, t_2 is firm productivity of a trading firm in an oligopoly, and \tilde{t}_1 and \tilde{t}_2 are the counterparts of non-trading firms. Aggregating these, we have

$$T_1 \equiv \int_0^{\tilde{z}\bar{z}} mt_1 dz = \frac{\tilde{z}\bar{z}\gamma L}{(1+\gamma)wf}, \quad \tilde{T}_1 \equiv \int_{\tilde{z}\bar{z}}^{\bar{z}} \tilde{m}\tilde{t}_1 dz = \frac{(1-\tilde{z})\bar{z}\gamma L}{(1+\gamma)wf} \quad (18)$$

$$T_2 \equiv \int_{\bar{z}}^{\bar{z}+\tilde{z}(1-\bar{z})} nt_2 dz = \frac{\tilde{z}(1-\bar{z})(2n-1)\gamma L}{2nwf}$$

$$\tilde{T}_2 \equiv \int_{\bar{z}+\tilde{z}(1-\bar{z})}^1 n\tilde{t}_2 dz = \frac{(1-\tilde{z})(1-\bar{z})(n-1)\gamma L}{nwf}, \quad (19)$$

where T_1 is the aggregate productivity of the trading industry under monopolistic competition, T_2 is that of the trading industry under oligopoly, and \tilde{T}_1 and \tilde{T}_2 are the counterparts for non-trading industries. Using these, we can obtain the productivity of the whole trading and non-trading industries as follows.

$$T_1 + T_2 = \frac{\tilde{z}\gamma L [2\bar{z}n + (1-\bar{z})(2n-1)(1+\gamma)]}{2n(1+\gamma)wf}$$

$$= \frac{\tilde{z}\gamma L [2n\bar{z} + (1-\bar{z})(2n-1)(1+\gamma)]}{(1+\gamma)f \{2n\bar{z} + (1-\bar{z})(1+\gamma)[\tilde{z} + 2(n-1)]\}} \quad (20)$$

$$\tilde{T}_1 + \tilde{T}_2 = \frac{(1-\tilde{z})\gamma L [n\bar{z} + (1-\bar{z})(n-1)(1+\gamma)]}{n(1+\gamma)wf}$$

$$= \frac{2(1-\tilde{z})\gamma L [n\bar{z} + (1-\bar{z})(n-1)(1+\gamma)]}{(1+\gamma)f \{2n\bar{z} + (1-\bar{z})(1+\gamma)[\tilde{z} + 2(n-1)]\}} \quad (21)$$

These expressions allow us to know how primitive parameters, e.g. L and \tilde{z} , affect the productivity of trading and non-trading sectors. Such effects are summarized in:

Proposition 2: The productivity of both trading and non-trading industries increases with market size. However, an increase in the portion of trading industries raises the productivity of the whole trading industry, but lowers that of the whole non-trading industry.

Proof. The effect of L is easily checked since (20) and (21) are increasing in L . On the other hand, differentiating (20) and (21) with respect to \tilde{z} yields

$$\begin{aligned}\frac{d(T_1 + T_2)}{d\tilde{z}} &= \frac{2\gamma L [n\bar{z} + (1 - \bar{z})(n - 1)(1 + \gamma)] [2n\bar{z} + (1 - \bar{z})(2n - 1)(1 + \gamma)]}{(1 + \gamma)f \{2n\bar{z} + (1 - \bar{z})(1 + \gamma)[\tilde{z} + 2(n - 1)]\}^2} > 0 \\ \frac{d(\tilde{T}_1 + \tilde{T}_2)}{d\tilde{z}} &= -\frac{2\gamma L [n\bar{z} + (1 - \bar{z})(n - 1)(1 + \gamma)] [2n\bar{z} + (1 - \bar{z})(2n - 1)(1 + \gamma)]}{(1 + \gamma)f \{2n\bar{z} + (1 - \bar{z})(1 + \gamma)[\tilde{z} + 2(n - 1)]\}^2} \\ &= -\frac{d(T_1 + T_2)}{d\tilde{z}} < 0,\end{aligned}$$

which establishes the latter half of the proposition. ||

The reason why the productivities increase with market size is simple but different between the monopolistically competitive and oligopolistic sectors. When L rises, total output of monopolistically competitive firms increases, and hence average cost and goods prices decline. In addition, Eq. (13) tells that lower goods prices expand the variety of differentiated goods. These effects jointly raise the productivity of monopolistically competitive industries. In contrast, an increase in L also improves the productivity of oligopolistic industries because it increases output of all firms, i.e. it has a pro-competitive effect.¹²

The effects of an increase in \tilde{z} are more complicated. On the one hand, the aggregate productivity of trading industries rises and that of non-trading industries declines as the first-order effect. On the other hand, as shown in Proposition 1, the wage rate rises with \tilde{z} , which has a negative impact on productivities. However, since the first-order effect is stronger than the indirect effect through the rise in wage rate, the trading industries' productivity improves. On the contrary, the non-trading industries' productivity necessarily decreases because both the direct and indirect effects explained above have a negative impact.

¹²The pro-competitive effect of trade between identical countries is, to our knowledge, shown first in Markusen (1981).

IV. WELFARE EFFECT

This section investigates the welfare effect of an increase in L and \tilde{z} . For this purpose, let us derive (per-capita) welfare W . Relating the market-clearing conditions of final goods to the utility function in (2) and rearranging terms, W is obtained as

$$\begin{aligned} W &\equiv \int_0^{\tilde{z}\tilde{z}} \ln \left[(2mx^\theta)^{\frac{1}{\theta}} \right] dz + \int_{\tilde{z}\tilde{z}}^{\tilde{z}} \ln \left[(\tilde{m}x^\theta)^{\frac{1}{\theta}} \right] dz \\ &\quad + \int_{\tilde{z}}^{\tilde{z}+\tilde{z}(1-\tilde{z})} \ln \left(\frac{ny}{L} \right) dz + \int_{\tilde{z}+\tilde{z}(1-\tilde{z})}^1 \ln \left(\frac{n\tilde{y}}{L} \right) dz \\ &= U_1 + U_2, \end{aligned} \quad (22)$$

where U_1 is utility from consuming monopolistically competitive products, and U_2 is utility from consuming oligopolized goods. Subutility U_1 can be rewritten as

$$\begin{aligned} U_1 &= \tilde{z}\tilde{z} \ln \left[(2m)^{\frac{1}{\theta}} x \right] + (1-\tilde{z})\tilde{z} \ln \left(\tilde{m}^{\frac{1}{\theta}} x \right) \\ &= \frac{\tilde{z}}{\theta} \ln \left\{ \frac{1}{w} \left[\frac{\gamma L x}{(1+\gamma)f} \right]^{\frac{\gamma}{1+\gamma}} \right\} + \tilde{z}\tilde{z} \ln (2^\gamma) - \frac{\tilde{z}(1-\theta)}{\theta} \ln x \\ &= \tilde{z}(1+\gamma) \ln \left[\frac{2n}{2n\tilde{z} + (1-\tilde{z})(1+\gamma)(\tilde{z}+2n-2)} \right] \\ &\quad + \tilde{z}\gamma \ln \left[\frac{\gamma L}{(1+\gamma)f} \right] + \gamma\tilde{z}\tilde{z} \ln 2, \end{aligned} \quad (23)$$

by substituting (12), (13) and (15). Similarly, U_2 has an alternative expression

$$\begin{aligned} U_2 &= \tilde{z}(1-\tilde{z}) \ln \left(\frac{ny}{L} \right) + (1-\tilde{z})(1-\tilde{z}) \ln \left(\frac{n\tilde{y}}{L} \right) \\ &= (1-\tilde{z}) \ln \left\{ n \left[\frac{\gamma L}{(1+\gamma)f} \right]^\gamma \left(\frac{1+\gamma}{n^2 w} \right)^{1+\gamma} \right\} \\ &\quad + (1-\tilde{z})(1+\gamma) \left[\tilde{z} \ln \left(\frac{2n-1}{2} \right) + (1-\tilde{z}) \ln(n-1) \right] \\ &= (1-\tilde{z}) \ln \left\{ \frac{1+\gamma}{n^{1+2\gamma}} \left(\frac{\gamma L}{f} \right)^\gamma \left[\frac{2n}{2n\tilde{z} + (1-\tilde{z})(1+\gamma)(\tilde{z}+2n-2)} \right] \right\} \\ &\quad + (1-\tilde{z})(1+\gamma) \left[\tilde{z} \ln \left(\frac{2n-1}{2} \right) + (1-\tilde{z}) \ln(n-1) \right], \end{aligned} \quad (24)$$

where use is made of (14) and (15). Summing (23) and (24) up, per-capita welfare is finally obtained as a function of parameters as follows.

$$\begin{aligned}
W = & (1 + \gamma) \ln \left[\frac{2n}{2n\bar{z} + (1 - \bar{z})(1 + \gamma)(\tilde{z} + 2n - 2)} \right] + \bar{z}\gamma \ln \left[\frac{\gamma L}{(1 + \gamma)f} \right] + \tilde{z}\bar{z}\gamma \ln 2 \\
& + (1 - \bar{z}) \ln \left[\frac{1 + \gamma}{n^{1+2\gamma}} \left(\frac{\gamma L}{f} \right)^\gamma \right] + (1 - \bar{z})(1 + \gamma) \left[\tilde{z} \ln \left(\frac{2n - 1}{2} \right) + (1 - \tilde{z}) \ln(n - 1) \right].
\end{aligned} \tag{25}$$

Based on these preparations, we now explore how an increase in L and \tilde{z} affects welfare. This is formally stated in:

Proposition 3: Welfare increases with market size, but it is unclear whether welfare increases with the proportion of trading industries.

Proof. Since W in (22) is monotonically increasing in L , the former part is proved. To see the effect of \tilde{z} , let us differentiate (22) with respect to \tilde{z} :

$$\begin{aligned}
\frac{dW}{d\tilde{z}} = & \bar{z}\gamma \ln 2 + (1 - \bar{z})(1 + \gamma) \ln \left(\frac{2n - 1}{2n - 2} \right) \\
& - \frac{(1 - \bar{z})(1 + \gamma)^2}{2n\bar{z} + (1 - \bar{z})(1 + \gamma)(\tilde{z} + 2n - 2)}.
\end{aligned} \tag{26}$$

The sign of (26) can be both positive and negative because the first two terms are positive but the last term is negative. ||

It is no surprise that welfare improves as the market size expands. The reason is that an increase in market size raises the product variety of monopolistically competitive goods and promotes competition, namely, reduces the goods price in the oligopolistic industries.

If the proportion of trading industries increases, there are two competing effects on welfare. First, noting that prices of tradables are lower than those of non-tradables, a rise in \tilde{z} tends to raise welfare by expanding the more efficient trading sectors. Second, as shown in Proposition 1, an increase in \tilde{z}

induces the equilibrium wage to rise. This raises the good prices of monopolistically competitive goods, and reduces the product variety. Simultaneously, the higher wage induced by an increase in \tilde{z} decreases output of oligopolistic firms, and raises the price of oligopolistic goods. Since the former effect has a positive effect on welfare and the latter effect has a negative effect, the total effect proves ambiguous.

Because it is analytically ambiguous whether welfare rises with \tilde{z} , we now resort to a numerical simulation. In order to neutralize the bias between the monopolistically competitive and oligopolistic industries, set $\bar{z} = 1/2$. And, let us set the other parameters as $\gamma = 1$, $n = 2$ and $L/f = 20$. Then, W in (25) is given by a function of \tilde{z} only:

$$W = 2 \ln \left(\frac{4}{\tilde{z} + 4} \right) + \frac{1}{2} \ln 10 + \frac{\tilde{z}}{2} \ln 2 + \frac{1}{2} \ln 5 + \tilde{z} \ln \frac{3}{2}.$$

The graph of W above is depicted by Figure 1 in the $\tilde{z} - W$ plane. It is clear that welfare is monotonically increasing in \tilde{z} , and hence we can conclude that welfare necessarily improves as a result of trade liberalization.

Figure 1 around here

V. DISCUSSION¹³

This section addresses four topics that are ignored in the previous sections. First, we briefly explain what follows if the whole industry is either monopolistically competitive or oligopolistic. Second, we consider the special but possibly realistic case in which trading industries are oligopolistic and non-trading industries are monopolistically competitive. Third, the difference in competitiveness between monopolistically competitive and oligopolistic firms is derived in our model. Finally, we relate our theoretical findings to the existing empirical evidence.

¹³This section is based on the referees' comments, which are gratefully acknowledged.

a) No coexistence of monopolistic competition and oligopoly

Thus far, we have focused on the case in which monopolistic competition and oligopoly coexist. We now address what follows if all industries are either monopolistically competitive or oligopolistic. If all industries are monopolistically competitive, an increase in market size L and the proportion of trading sectors \tilde{z} necessarily improves welfare for the following reason. If L rises, total output of all firms increases, and the price of all goods falls while the equilibrium wage remains unchanged. Hence, the price index of consumers decreases, and welfare rises. If \tilde{z} rises, there are two effects. The first is the effect of increasing the traded variety, which has a positive effect on welfare. The second is the effect of reducing the wage rate, which lowers the product price of all varieties and raises welfare.

In contrast, the results differ if all industries are oligopolistic.¹⁴ In this case, an increase in L improves welfare, but the welfare effect of an increase in \tilde{z} is unclear for the following reason. If L rises, all firms in both the trading and non-trading industries increase output, but the equilibrium wage is not affected. Therefore, the price of both traded and non-traded goods declines, and welfare improves. However, the effect of an increase in \tilde{z} is more complex. On the one hand, an increase in \tilde{z} promotes competition among trading firms, tending to raise welfare. On the other hand, the wage rate rises with \tilde{z} , which, in turn, raises the price of both traded and non-traded goods, and tends to lower welfare. As a result, it is ambiguous whether an increase in \tilde{z} improves welfare when oligopoly prevails in all industries.

¹⁴This case is examined in details in Fujiwara and Kamei (2018).

b) Oligopoly in trading industries and monopolistic competition in non-trading industries

As is noted in Introduction, growing evidence suggests that a small number of firms engage in exporting.¹⁵ Thus, it is natural to ask what happens when trading industries are oligopolistic and non-trading industries are monopolistically competitive. In this special case, the per-capita utility function can be re-defined as

$$u = \int_0^{\tilde{z}} \ln \tilde{X}_1(z) dz + \int_{\tilde{z}}^1 \ln X_2(z) dz,$$

where $\tilde{X}_1(z)$ is the quantity index of non-traded goods (monopolistically competitive goods), and $X_2(z)$ is consumption of traded goods (oligopolized goods), where $\tilde{X}_1(z)$ is defined in Eq. (3). Note that in the present case, an expansion of trading industries is modeled by a *decrease* in \tilde{z} . By solving the model similarly to the original model, the equilibrium wage is obtained as

$$w = \frac{[1 - (2n - 1)\gamma]\tilde{z} + (2n - 1)(1 + \gamma)}{2n}.$$

Differentiating this with respect to \tilde{z} yields

$$\frac{\partial w}{\partial \tilde{z}} = \frac{1 - (2n - 1)\gamma}{2n},$$

the sign of which is unclear. As a natural consequence, the welfare effect of a decrease in \tilde{z} is unclear.

However, a numerical simulation enables us to know an interesting relationship between per-capita welfare W and \tilde{z} . To see this, let us derive the closed form of W . Tedious manipulations lead to

$$W = (1 + \gamma) \ln \left[\frac{(2n - 1)(1 + \gamma)}{n \{ [1 - (2n - 1)\gamma]\tilde{z} + (2n - 1)(1 + \gamma) \}} \right] + \gamma \ln \left[\frac{\gamma L}{(1 + \gamma)f} \right] + (1 - \tilde{z}) \ln n.$$

While this is a complicated function of parameters, setting $\gamma = 1$, $n = 2$ and $L/f = 20$ yields

$$W = 2 \ln \left(\frac{3}{6 - 2\tilde{z}} \right) + \ln 10 + (1 - \tilde{z}) \ln 2.$$

¹⁵See the subsection d) for further empirical studies.

The graph of W above is given by Figure 2 in the $\tilde{z} - W$ plane.

Figure 2 around here

It is interesting that a reduction in \tilde{z} raises (resp. lowers) welfare if \tilde{z} is smaller (resp. larger) than about 0.1. This implies that trade liberalization in the form of increasing \tilde{z} is welfare-enhancing only if the country is sufficiently open (\tilde{z} is low enough).¹⁶

c) Competitiveness

Thus far, we have not discussed the competitiveness of monopolistically competitive and oligopolistic firms. However, it is useful to examine how the competitiveness differs between monopolistically competitive and oligopolistic industries. For this purpose, we define the competitiveness by an inverse of the markup (price divided by marginal cost).¹⁷ Then, some manipulations lead to

$$\begin{aligned} \text{Competitiveness of monopolistically competitive firms} &= \theta \\ \text{Competitiveness of oligopolistic firms} &= \frac{2n-1}{2n}, \end{aligned}$$

for trading firms.¹⁸ These results are intuitively natural. Since the elasticity of substitution is $1/(1-\theta)$ and increases with θ , higher θ means higher substitutability among differentiated products, and hence the goods price approaches marginal cost, i.e. the competitiveness rises. Similarly, as n increases, each oligopolistic firm produces more, and hence the goods price converges to marginal cost and the competitiveness rises. Given these results,

¹⁶The same result is analytically obtained in Kreickemeier and Meland (2013) even though the model is quite different.

¹⁷Nothing changes even if the inverse of the Lerner Index $((p - MC)/p)$ is used, where MC denotes marginal cost.

¹⁸The counterparts of non-trading firms are respectively θ (monopolistically competitive firms) and $(n-1)/n$ (oligopolistic firms).

we can say that monopolistically competitive firms are more competitive than oligopolistic firms if and only if $\theta > (2n - 1)/2n$ for the trading industries and $\theta > (n - 1)/n$ for the non-trading industries.

d) Relation to empirics

While the monopolistic competition model with massless firms dominated in trade theory over the last decade, recent evidence suggests the substantial role of large firms in international trade.¹⁹ Introducing several empirical works, this subsection discusses how our theory is related to them.

As introduced in Introduction, Bernard et al. (2007) and Mayer and Ottaviano (2008) are two of the earliest papers that empirically find that exports are highly concentrated on a small number of large firms.²⁰ Given this fact, some papers employed an oligopoly model to study the effect of trade liberalization quantitatively. Atkeson and Burstein (2008) show that incomplete pass-through and pricing-to-market are well explained in a Cournot model rather than the monopolistic competition model. Edmond et al. (2015) apply the Atkeson-Burstein model to examine the welfare effect of trade liberalization, and find that the opening of trade of Taiwan reduces the product market distortions by about 1/5, i.e. the pro-competitive gains from trade are significant. Considering China's WTO accession in 2001, Lu and Yu (2015) also find that trade liberalization reduces markup dispersion. Selecting seven industries, Sutton and Trefler (2016, p. 829) find that 'just four firms in each industry produce between 21 percent and 70 percent of global output.' Using the data of India from 1989 to 1997, De Loecker et al. (2016) empirically demonstrate that that trade liberalization exerted pro-competitive pressure on markups. All of these previous works suggest the usefulness of the oligopoly model in analyzing international trade today and supplements

¹⁹Head and Spencer (2017) review the recent revival of oligopoly models of international trade in relation to empirical evidence.

²⁰Bernard et al. (2018) provide a comprehensive account of global firms theoretically and empirically.

the monopolistic competition model.

Furthermore, Hottman et al. (2016) find a more interesting result. Using barcode data, they show that half of output in a product group is produced by five firms and that 98% have market share less than 2%. Their result clearly suggests that both large oligopolists and small monopolistically competitive firms coexist, and thus providing an empirical support for the theoretical analysis in Shimomura and Thisse (2012), Parenti (2018) and Vavoura (2017). Although our treatment is different from these authors in the sense that we are considering the coexistence of monopolistically competitive and oligopolistic *industries*, the evidence of Hottman et al. (2016) may support our model.

VI. CONCLUSIONS

We have considered some implications for international trade of a coexistence of monopolistic competition and an oligopoly, which is a recent topic in industrial organization. While the previous works assume a coexistence of two kinds of *firms*, we suppose a coexistence of two kinds of *industries* by utilizing Neary's (2003, 2016) approach. Then, we have shown that an increase in market size raises the productivity of all industries and welfare. But, if the share of trading sectors rises, the productivity of trading industries improves, but that of non-trading industries worsens. However, our numerical simulation ensures a welfare improvement from an increase in the proportion of trading industries.

We believe that our results shed light on the ongoing debate on globalization, but recognize that a number of limitations remain. First, we have assumed that \tilde{z} is exogenous, following the existing literature on the GOLE model. This simplifies analysis, but the reallocation effect of Melitz (2003) is excluded. Given the theoretical and empirical importance of the reallocation effect, it is needed to reexamine the effect of trade liberalization by endogenizing \tilde{z} . Second, trade barriers such as transport costs and import tariffs

are assumed way. This is because they extremely complicate analysis, and nothing clear is obtained. However, in view of the reality that reductions in transport costs and/or import tariffs are a driving force for expanding world trade, it is needed to incorporate them. Third, our results hinge on the specific functional forms, e.g. logarithmic utility, and many other restrictions. Finally, we have made no empirical analysis. Further research is called for so as to take into account these limitations.

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Figure 1: Welfare and tilde z

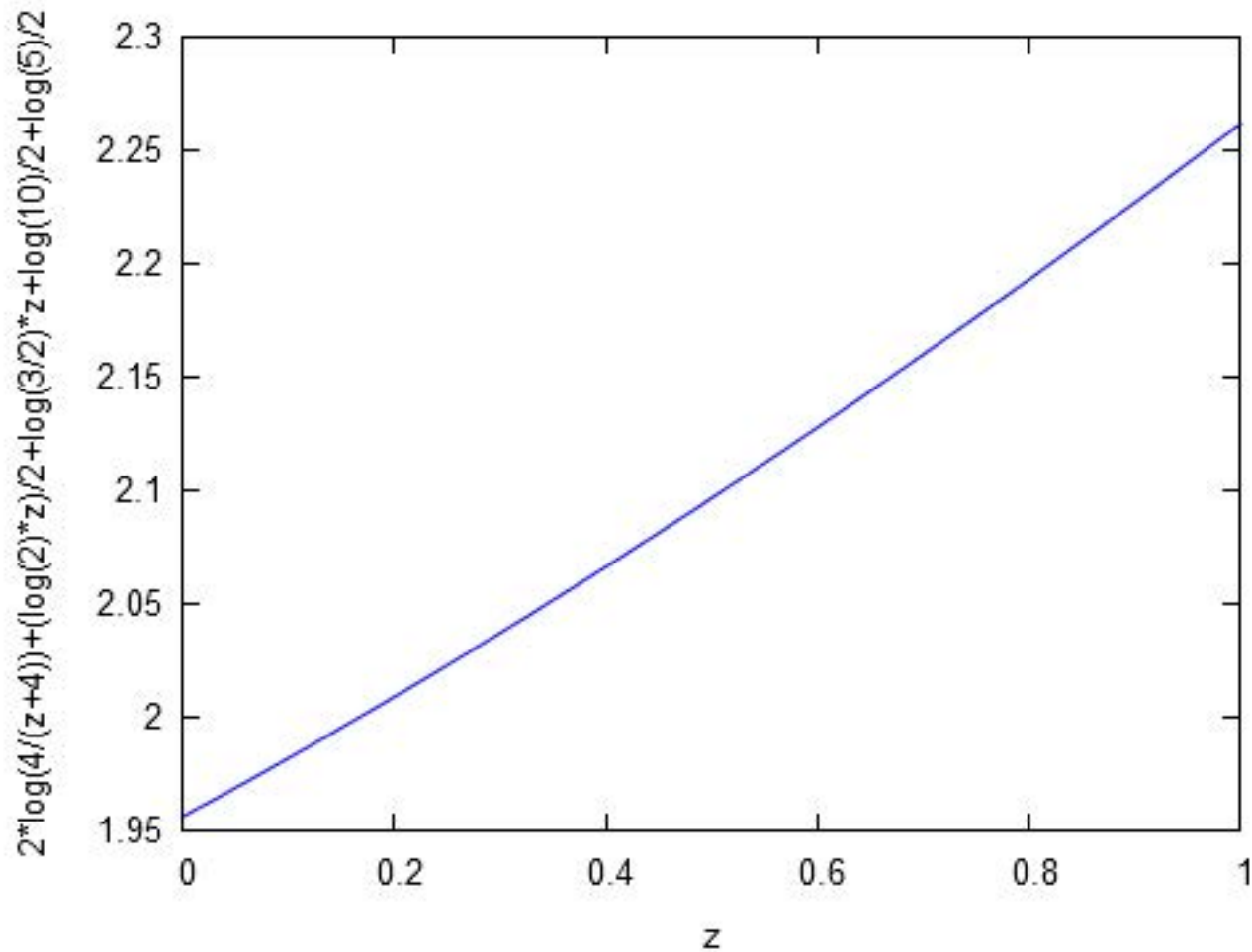


Figure 2: Welfare and tilde z

